On the Scattering of β-Rays.

By J. A. CROWTHER, Sc.D., F.Inst.P., University Lecturer in Physics as applied to Medical Radiology, Cambridge, and B. F. J. SCHONLAND, O.B.E., B.A., George Green Student of Gonville and Caius College, Cambridge.

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Introduction.

If a parallel pencil of homogeneous β-rays is made to fall on a thin sheet of any material the individual β-particles are deflected from their original path and the beam becomes scattered or diffuse. The deflections may be very considerable, even when the thickness of the material is far too small to produce any appreciable absorption either in the number or the energy of the particles in the beam. These deflections are generally ascribed to the action of the strong electric fields within the atoms of the scattering material, and measurements on the scattering of the rays afford a useful means of investigating the nature and magnitude of these atomic fields.

Experiments on the scattering of a beam of homogeneous β-rays were made some years ago by one of the writers* with the object of obtaining an estimate of the number of electrons in the atoms of different elements. A theory of the phenomenon had been previously put forward by Sir J. J. Thomson† on the assumption, which was then generally accepted, that the atom consisted of a number, N, of negative electrons embedded in a sphere of positive electrification of atomic radius. On this view the deflection of a β-particle may be due either to the repulsion of the negative electrons in the atom, or to the attraction of the positive sphere. The probability of a particle suffering a large deflection from either of these causes could be shown to be negligibly small, and the actual observed deflections to be due to the cumulative effects of a large number of small deflections. Applying a theorem due to Lord Rayleigh it was shown that the probability that the deflection of the particle should be less than a given angle ϕ was equal to $1-\epsilon^{-\phi^2/kt}$ where t is the thickness of the scattering material, and k a constant which depends on the nature of the substance and the energy of the rays. Thus, if the rays after passing through the scattering substance were limited by a circular stop, whose radius subtended an angle ϕ at the substance, the intensity of the radiation passing through the stop should be given by

$$I/I_0 = 1-\epsilon^{-\phi^2/kt}, \tag{1}$$

where I_0 is the initial intensity of the beam.

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* J. A. Crowther, 'Roy. Soc. Proc.,' A, vol. 84, p. 226 (1910).

† Sir J. J. Thomson, 'Camb. Phil. Soc. Proc.,' vol. 15, p. 465 (1910).

If t_m is the thickness of the scattering foil necessary to cut down the radiation through the stop to half value, then for a given substance and given radiation

$$\phi^2/t_m = \text{constant}. \tag{2}$$

It was further shown that if T is the kinetic energy of the individual β-particle, e the charge on an electron, N the number of electrons in the atom, and n the number of atoms per cubic centimetre of the scattering material,

$$\phi = \frac{e^2}{mv^2}\left\{\frac{384}{25}\,\mathrm{N}_0+\frac{\pi^2}{16}\,\mathrm{N}^2\right\}^{\frac{1}{2}}\sqrt{\pi n t_m}. \tag{3}$$

In the following year Sir Ernest Rutherford* published a new theory of atomic structure in which he suggested that the positive charge in the atom, instead of occupying a sphere of atomic radius, was in reality confined to a nucleus of exceedingly small dimensions, so small, in fact, that it could be regarded as acting as a point-charge. This change of view was necessary to account for the very large deflections occasionally experienced by α-particles during their passage through matter, and which the Thomson atom was incapable of explaining. The field due to such a central charge would produce deflections both in the α- and the β-rays in comparison with which the deflections produced by the electrons in the atom would be negligible. The measured deflections in this case would be the large deflections produced by encounters with the positive nucleus of the atom. It was shown that until the material was so thick that half the particles had suffered one such deflection, the probability of a particle suffering two such deflections was negligibly small. The scattering was thus the result of single encounters of the particles with the nucleus of the atom. This may be called "single" scattering, to distinguish it from the multiple or "compound" scattering of Thomson's theory.

The probability that a particle shall suffer a deflection greater than ϕ is, on the "single" scattering hypothesis, obviously proportional to the thickness of material it has to traverse. Thus the probability that the particle will still be included within an angle, ϕ, after passing through a thickness, t, of the substance is $1-kt$, or

$$\mathrm{I}/\mathrm{I}_0 = 1-kt. \tag{4}$$

It can be shown that ϕ^2/t_m is constant on the theory of single scattering. The experimental distinction between "single" and "compound" scattering lies, therefore, in the different relations given by the two for the variation in the intensity of the radiation through a fixed stop with increasing thickness of the scattering material. For single scattering the relation (4) is linear, for compound scattering it is a curve given by (1) with a double inflection.

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* Sir E. Rutherford, 'Phil. Mag.' [6], vol. 21, p. 669 (1911).

As the conditions for single scattering must ultimately cease to hold for thick foils (according to Rutherford's theory it holds until the radiation has been reduced to about half value) the transmitted radiation is bound eventually to decrease less rapidly than the linear relation, and the curve to become asymptotic to the axis of t. The possibility of discriminating between the two theories experimentally thus depends on the accuracy with which the initial portion of the curve can be determined. The experiments already referred to appeared to be in agreement with the theory of Sir J. J. Thomson, and values for N were deduced from (3) which showed that the number of electrons per atom was three times the atomic weight. The author's results were recalculated by Rutherford, on the nuclear theory, and gave values for N which were somewhat less than the atomic weight. The actual number of electrons per atom is now known with certainty to be equal to the atomic number of the elements, *i.e.,* approximately half the atomic weight. It was felt that, while the divergences between the experimental curves and relation (4) might be due to experimental imperfections, the differences between the calculated values of N and the corresponding atomic number were too large to be accounted for in this way. We were, therefore, very glad of the opportunity afforded by Sir Ernest Rutherford's very kind offer of a supply of radium emanation to investigate the matter in a more accurate way than had been possible with the comparatively weak source originally available.

The centre of interest had, of course, changed in the interval which had elapsed since the first paper was published. The nuclear theory of the atom is now well established, and the recent experiments of Chadwick, supported by a great mass of indirect evidence of many kinds, have proved that the charge on the nucleus, and, therefore, the number of electrons per atom, is exactly equal to the atomic number of the element. The discrepancies between the theory and the experiments on the scattering of β-rays, if proved to be genuine, would, therefore, be due to unexpected variations in the forces within the atom, and might throw new light on this very interesting point.

Experimental Details.

In principle the method of experiment was identical with that of the previous experiments, but many modifications have been made, suggested by experience and made possible by the greater intensity of the source, which have greatly increased the accuracy of the results. The present form of the apparatus is shown in fig. 1.

The source of the β-radiation is radium emanation contained in a thin-walled glass bulb of about 3 mm. diameter and placed at R. After passing through a stop, a_1, of 3 mm. diameter, the rays are formed into a magnetic spectrum by passing between the poles of an electromagnet, the magnetic field being at right angles to the plane of the diagram. The path of the rays which actually reach the scattering material at P is defined by a series of stops, a_1, a_2, a_3. It will be seen that they have described a semicircle in the magnetic field, the radius of which in the actual experiments was 2·47 cm. The velocity of the β-particles employed can thus be determined from the applied magnetic field and the radius of the path. Calculation showed that with the apertures actually employed the extreme velocities of the emerging β-particles did not differ from the mean velocity by more than about 1 per cent. This 180° deflection of the rays, though not without its technical difficulties, produces a beam which is greatly superior in every way to that produced by the 90° deflection employed in the previous experiments.

On leaving the magnetic field the rays pass directly down the axis of a thick-walled iron tube, and are further screened from the magnetic field by large blocks of soft iron. After passing through another stop, a_4, also of 3 mm. diameter, the pencil falls normally on the scattering substance at P. In practice a number of these substances (or a number of different thicknesses of the same substance) were mounted on a frame moving in a fixed slide and operated from outside the chamber by a windlass, so that each in turn could be placed in the path of the rays. The diverging pencil produced by scattering in the substance at P could be limited by stops of different diameter placed at S. The intensity of the radiation passing through the stop was measured by means of an ionisation chamber, J.

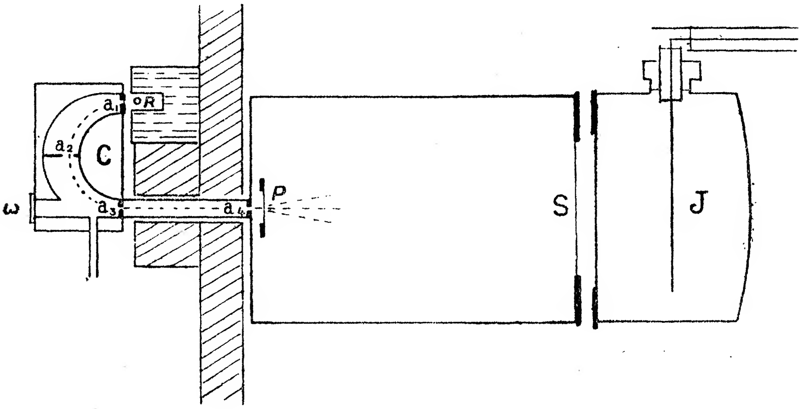

FIG. 1.

In order to eliminate any scattering of the rays by air along their course, the apparatus was made airtight, the apertures a_1 and S being closed by thin aluminium windows. The apparatus was filled with hydrogen and then exhausted by a water pump. Experiment showed that, under these circumstances, there was no perceptible radiation outside the geometrical limits of the beam in the absence of any scattering substance at P. There was thus no scattering of the radiation by the residual gas, and no appreciable reflection of the primary radiation from the walls of the apparatus.

The ionisation produced in J was measured by a balance method, using the uranium oxide compensator, which has been fully described in previous papers.* The compensator was again calibrated during the course of the present experiment, and the readings of the screw were found to be exactly proportional to the ionisation over the whole range employed. The position of balance was determined by a Wilson tilted electroscope, working at a sensitivity of about 90 divisions per volt. The connecting tubes were filled with sulphur, to reduce the loss of charge owing to ionisation of the air by γ-radiation.

The apparatus was screened as far as possible from the γ-radiation from the tube R by blocks of lead, but there was a very considerable residual effect. As the emanation decays to half value in 3·85 days, it was considered advisable to balance out the γ-ray effect by means of an auxiliary tube containing a small quantity of emanation placed near the compensator. The two emanation tubes were filled at the same time, and the balance when once established was thus automatically maintained. It may be worth mentioning in passing that it was found impossible to obtain a steady balance between the two tubes until some 10 hours had elapsed after filling the tubes.

The source, R, had generally an initial activity equivalent to that of 45 mgrm. of radium. The intensity of the β-radiation from this produced an ionisation in J which required eight complete turns of the compensator screw for compensation. A movement of the compensator screw of one-twentieth of a turn produced a sensible disturbance of the balance. Readings could thus be made to an accuracy of well within 1 per cent. of the initial intensity of the beam. The accuracy attainable is of course diminished as the emanation decayed. The source was generally replaced by a fresh one when its activity had fallen to about half value. The gradual decay in intensity of the β-radiation, which amounted to less than 1 per cent. per hour, caused no difficulties in obtaining a balance. Readings made at different times could easily be reduced to a standard value, as the radioactive constant of the emanation is known to an accuracy of at least 0·1 per cent.

In making the observations, the magnetic field was first applied so as to deflect the β-rays away from the scattering chamber, and an accurate balance obtained for the γ-ray effect alone. The field was then reversed, so as to deflect the rays on to the scattering substance, and a balance again obtained. The difference between the two compensator readings measured the intensity of the β-radiation transmitted through the stop. As a further precaution, both the balances were again verified. The process is tedious, the determination of a single point generally taking about $1\frac{1}{2}$ hours. It is, however, by far the most accurate method of measuring small changes in an ionisation current, and readings could be repeated on different occasions to an accuracy of at least 1 per cent.

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* J. A. Crowther, *loc cit.*

The scattering substances consisted of the thinnest foils of the elements concerned. Their thickness was measured by finding the mass of a known area of the foil. The average thickness was thus determined to an accuracy of 1 per cent. It is possible, however, that individual pieces of the foil might differ from the mean thickness, but the experimental results gave no evidence of this, although readings were made with different portions of the foil to test this point.

The magnetic field was measured by means of a fluxmeter, which was itself calibrated by means of a Hibbert standard, and also by a standard field employed by Mr. C. D. Ellis, and was found to be accurate to within 1 per cent. The kinetic energy, T, of the β-rays was calculated from the magnetic field and radius of curvature of the path, using the Planck-Einstein relation

$$\mathrm{T} = \mathrm{V}e = m_0c^2\left[\frac{1}{\sqrt{1-\beta^2}}-1\right], \tag{5}$$

V being the potential differences necessary to give the electron the energy T. Following the usual custom, the energy of the β-particle in this paper is given in terms of V, measured in volts.

Variation of Scattering with Thickness of Material.

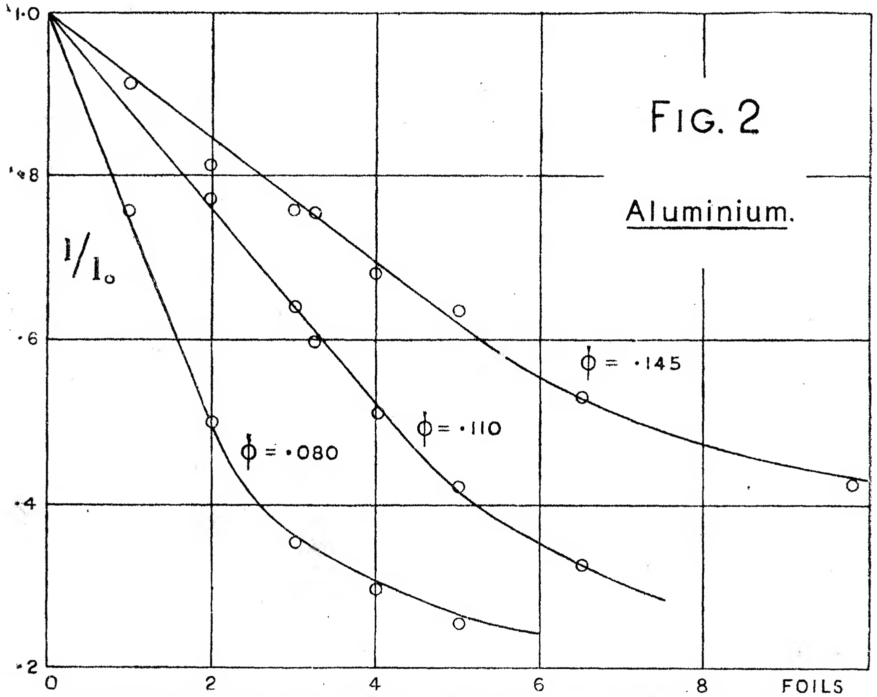
The first point to be investigated was that of the relation between the intensity of the radiation transmitted through a stop of given aperture and the thickness of the scattering material. Bundles of one, two, three, four and five thicknesses of the same foil, cut from the same sheet, were mounted on the slide, and the intensity of the rays transmitted through the stop was measured for each of these bundles in turn. Experiments were made with stops subtending four different angles at the scattering substance, and both aluminium and gold were used in turn as the scattering substance. Some of the results are shown graphically in figs. 2 and 3, and specimens of the actual readings are given in Table I.

It will be seen that, in every case, the curve showing the relation between the transmitted radiation and the thickness of the absorbing material is a straight line, until the radiation has fallen to at least half value. The intensity is thus given by

$$I/I_0 = 1-kt, \tag{4}$$

and not by equation (1). The scattering of the β-particles in the foils used in these experiments is therefore " single " scattering and not " compound."

It will be seen that the initial flattening of the curve, which was so noticeable in the curves obtained in the previous experiments, and which is the criterion of compound scattering, is entirely absent from the present curves. The departure of the original curves from the linear relation may be ascribed almost certainly to the fact that, owing to the small intensity of the source, it was necessary to use a beam of small, but still finite, divergence.



Thickness of 1 foil, 8·45 . 10^{-5} cms. Energy of radiation, 4·59 . 10^5 volts.

The matter could be tested experimentally by replacing the four stops by stops of larger aperture, say 5 mm. diameter, or more simply still by throwing the beam slightly out of alignment, so that its axis passed a little to one side of the centre of the stop S. Experiments were made under these conditions, and the experimental curves obtained always showed an initial flattening, sometimes in an exaggerated form.

These observations showed the importance of using a pencil of rays as nearly parallel as circumstances would permit. On the other hand, the intensity of the transmitted beam of radiation falls off very rapidly as the aperture of the stops is reduced. The stops actually used in the measurements recorded had a diameter of 3 mm. These stops allowed sufficient radiation to be transmitted to permit its intensity to be measured to an accuracy of about 1 per cent., and at the same time, provided a pencil of sufficiently small divergence to render the error due to the divergence of the beam negligible, at any rate in the case of the larger stops. Since any error in centring the beam would produce similar effects to those produced by a divergence of the beam itself, the importance of the utmost accuracy in centring is apparent. The centring in these experiments was effected by sighting through the two stops, a_3 and a_4, on the intersection of a pair of fine

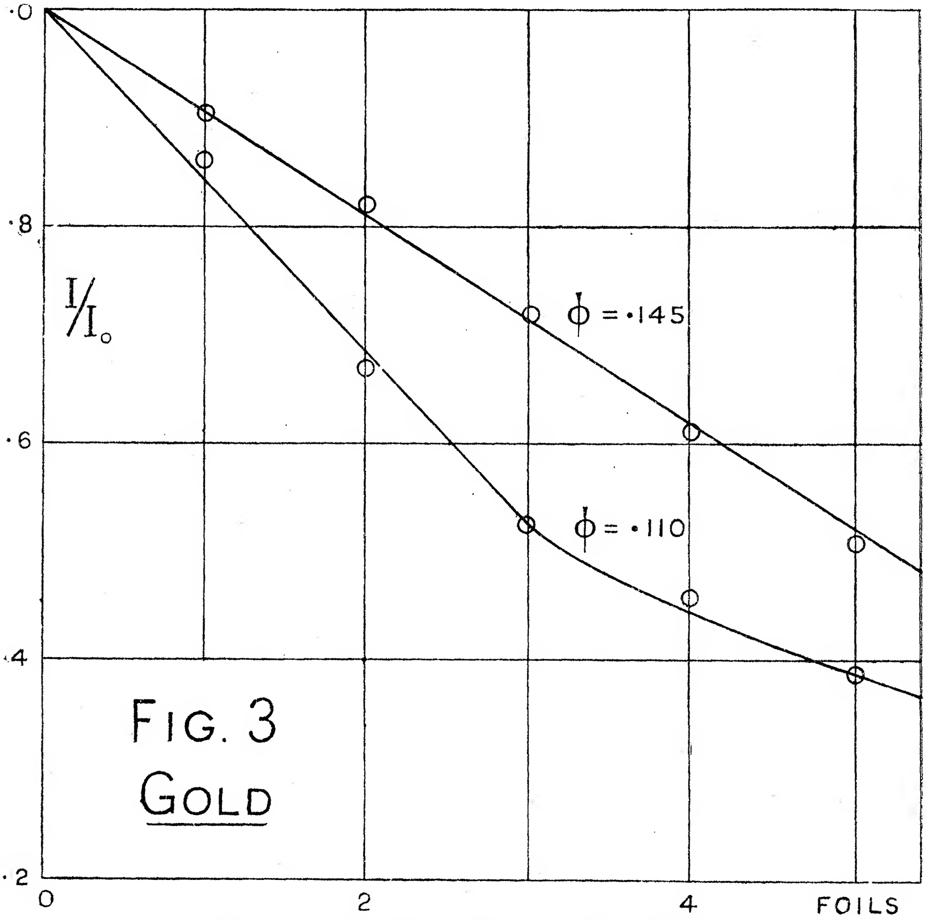


FIG. 3
GOLD

1 foil = 8·48 . 10^{-6} cms. Energy = 4·59 . 10^5 volts.

Table I.—Typical Set of Measurements.

June 2, 1921. Gold: Thickness of one foil = 8·48 . 10^{-6} cm. ϕ = 0·145 radians. Energy of rays 4·59 . 10^5 volts.

No. of foils.	Reading for γ ray balance.	Time.	Reading for $\beta+\gamma$ ray balance.	Time.	Difference (β rays) I.	Time from Foil 0 (t).	$e^{-\lambda t}$.	$I/I_0e^{\lambda t}$.
0	2·670 2·665	9.40 A.M. 11.0 ,,	2·990 —	10.40 A.M. —	0·320 0·325 } 0·323	0 (10 A.M.)	1	1
1	2·540 2·540	11.20 ,, 12.0 P.M.	3·823 3·823	11.50 ,, 12.10 P.M.	} 0·283	2·0 hours	0·985	0·900
2	2·528 2·515	12.20 ,, 12.50 ,,	2·780 —	12.35 ,, —	0·252 0·265 } 0·258	2·5 ,,	0·981	0·822
3	2·460 2·460	1.45 ,, 2.25 ,,	2·685 —	2.15 ,, —	} 0·225	4·0 ,,	0·970	0·722
4	2·444 2·444	2.45 ,, 3.35 ,,	2·618 2·623	3.10 ,, 3.30 ,,	0·174 0·179 } 0·176	5·25 ,,	0·962	0·580
5	2·392 2·392	3.50 ,, 4.45 ,,	2·545 2·545	4.15 ,, 4.30 ,,	0·153 —	6·5 ,, —	} 0·953	0·503

wires, intersecting exactly at the centre of the stop S. For this purpose, a glass window, *ω*, was made in the deflection chamber, C. In this way it was easy to ensure that axis of the beam was not more than 0·2 mm. from the centre of the stop.

The effect of a diverging beam on the intensity of the radiation transmitted through the stop can easily be demonstrated from general considerations. Let ABCD (fig. 4*a*) be the stop S, and with centre at O, and let P be a point within the cross-section of the unscattered beam. Describe a circle, AQCR, round P as centre, with radius equal to the radius of the stop. The area AQCB is equal to the area ADCR. If P had been at O, the whole of the radiation within the circle AQCR would have passed through the stop. Owing to the divergence of the pencil, the radiation through the area AQCB is cut off by the stop, but the radiation through the area ADCR is transmitted in its place. The error due to divergence of the beam is measured by the difference in the radiation through these two areas. This will depend on the distribution of the scattered radiation about the axis of the beam.

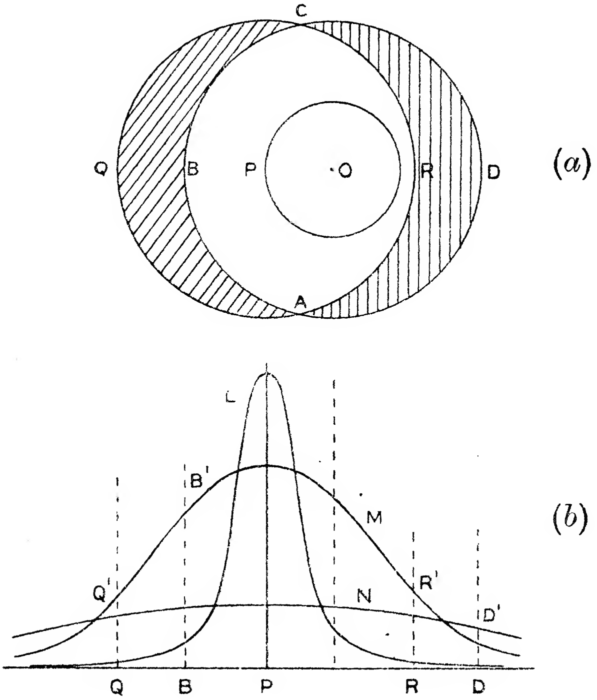


Fig. 4.

We may assume that this distribution can be represented by probability curves, as indicated in fig. 4*b*, the width of the curve increasing as the thickness of the scattering substance, and hence the most probable angle of scattering increases. Thus L represents the distribution curve for a very thin sheet of material, M for a greater thickness, and N for a still greater thickness. If PB, PQ, ... on this diagram represent the angles subtended at the scattering substance by the lines PB, PQ, ... in the other diagram, the error due to the obliquity of P will be the difference between the areas cut off from the curve between the ordinates at Q and B, and those at R and D, the area QBB′Q′ being lost, and the area RDD′R′ being added to the radiation through the stop. Now for very thin sheets of the scattering material (Curve L) this difference is negligible, and the initial portion of the curve will be practically unaffected by the obliquity of P. As the thickness of material increase (Curve M) the radiation lost becomes appreciably greater than that gained, and the experimental curves will fall appreciably below those for a narrow parallel pencil. The divergence of the beam produces a deviation from the linear relationship corresponding very closely with that observed in the earlier experiments. A consideration of curve N will show that the error becomes negligible again for thick foils.

We conclude then that the relation between the transmitted radiation and the thickness of the scattering material is a linear one for thin foils, the linear relation holding until approximately half the radiation has been deflected outside the stop. The results of the experiments with different materials and stops of different angle can be most conveniently recorded by giving the thickness, t_m, of material required to cut down the radiation through the stop to one half its initial intensity. In a few cases where the experimental curves seemed to deviate slightly from the linear relation before the intensity had been reduced to half, the half value thickness actually given in the Table of Results was obtained by producing the straight portion of the curve. The correction was, however, never more than a few per cent.

Collected Results.

The results obtained are summarised in Tables II and III. The whole of the observations in Table II were made with a deflecting magnetic field of 1110 gauss ($HR = 2740$ gauss-cm.), the corresponding velocity of the rays being $2{\cdot}552 \times 10^{10}$ cm./sec., and the energy of the particles $4{\cdot}59 \times 10^5$ volts. The first column indicates the nature of the scattering substance, together with its atomic number and atomic weight, the former being indicated by the clarendon type. Column II gives the angle of scattering, that is to say the angle subtended by the radius of the stop at the scattering substance. The third column gives the mass per unit area, x_m, of the foil required to cut down the radiation transmitted through the stop to half value. As the thickness of the

Table II.

$H\rho = 2740$ gauss-cm. $\beta = 0{\cdot}851$. $T/e = 4{\cdot}59 \,.\, 10^5$ volts.

I.	II.	III.	IV.	V.		VI.		VII.	VIII.
Substance.	ϕ (radians).	x_m (grm./cm.^{2}).	t_m (cm.).	ϕ^2/t_m.	$\phi/\sqrt{t_m}$.	ϕ^2/x_m.	$\phi/\sqrt{x_m}$.	$\phi\sqrt{A/x_m}$.	$\phi\sqrt{A/x_m} \times 1/N$.
Al **13** 27·1	0·145	1·65 × 10⁻³	6·00 × 10⁻⁴	35·0	5·92	12·7	3·57	18·6	1·43
	0·110	0·906 × 10⁻³	3·42 × 10⁻⁴	35·3	5·94				
	0·094	0·672 × 10⁻³	2·54 × 10⁻⁴	34·7	5·89				
	0·0803	0·505 × 10⁻³	1·90 × 10⁻⁴	34·0	5·83				
Cu **29** 63·6......	0·145	1·67 × 10⁻³	1·86 × 10⁻⁴	113	10·6	12·6	3·55	28·3	0·976
	0·110	0·945 × 10⁻³	1·05 × 10⁻⁴	115	10·7				
Ag **47** 107·9 ...	0·145	1·16 × 10⁻³	1·11 × 10⁻⁴	189	13·75	18·1	4·25	44·2	0·940
	0·110	0·706 × 10⁻³	0·675 × 10⁻⁴	179	13·4				
Au **79** 197·2 ...	0·145	0·843 × 10⁻³	4·37 × 10⁻⁵	480	21·9	24·9	4·99	70·0	0·886
	0·110	0·507 × 10⁻³	2·63 × 10⁻⁵	460	21·5				
Celluloid **7** 14·5	0·145	3·17 × 10⁻³	—	—	—	6·60	2·57	9·96	1·42
	0·110	1·71 × 10⁻³	—	—	—	7·10			
Carbon **6** 12 ...	0·145	2·56 × 10⁻³	—	—	—	8·20	2·86	9·9	1·65
Rice paper	0·110	1·73 × 10⁻³	—	—	—	7·0			

Values in Columns VII and VIII for celluloid calculated from the formula $C_{12}H_{18}N_2O_{16}$, neglecting hydrogen. Thus mean A.W. = 14·5, and mean atomic No. = 7. Values for carbon obtained from carbonised paper.

Table III.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
Element.	ϕ (radians).	Hρ.	E volts (T/e).	t_m (foils).	$\phi/\sqrt{t_m}$.	$f^2(\phi,\beta)$.	$\frac{1}{2}mv^2/e$.	$\phi/\sqrt{t_m}\times\frac{1}{2}mv^2/e$.	$\phi/\sqrt{t_m}\times$ E.	$\phi/\sqrt{t_m}\times$ E$/f(\phi,\beta)$.
Au	0·110	4842	$10·3\times10^5$	13·3	$3·02\times10^{-2}$	2·41	$13·71\times10^{13}$	$4·19\times10^{12}$	$3·11\times10^4$	$2·00\times10^4$
	0·145	4101	8·23	15·3	3·71	2·34	11·37	4·22	3·05	1·99
	0·145	2742	4·59	5·15	6·39	1·92	6·97	4·45	2·93	2·11
	0·145	1694	2·11	1·70	11·1	1·45	3·59	3·99	2·34	1·94
Al	0·110	4842	10·3	14·3	2·91	2·41	13·71	3·99	3·00	1·93
	0·110	2742	4·59	4·05	5·46	1·81	6·97	3·81	2·51	1·86
	0·110	1870	2·51	2·31	7·24	1·28	4·15	3·01	1·81	1·60
C	0·110	4842	10·3	4·55	5·16	2·41	13·71	7·08	5·32	3·42
	0·110	2742	4·59	1·29	9·70	1·81	6·97	6·76	4·45	3·30
	0·145	2742	4·59	2·40	9·36	1·92	6·97	6·53	4·30	3·10
	0·145	1694	2·11	0·87	15·5	1·45	3·59	5·56	3·27	2·71

foils was always determined by weighing, x_m is the quantity actually measured. The actual thickness, t_m, given in Column IV is deduced from Column III, assuming the ordinary values for the density of the element. Column V gives the corresponding values of $\phi/\sqrt{t_m}$ and ϕ^2/t_m, and Column VI those of ϕ^2/x_m and $\phi/\sqrt{x_m}$. The final column, VIII, gives the values of $(\phi/\mathrm{N})\sqrt{\mathrm{A}/x_m}$ where A is the atomic weight and N the atomic number. The significance of this column will be discussed later.

The value of the field in the experiments in Table II was chosen as giving the maximum intensity of β-radiation. Table III records the results of experiments made with rays of other velocities. The initial intensity of this radiation was generally small, and although this was partly counteracted by using a source of nearly twice the usual intensity the results cannot be relied on to the same order of accuracy as those in the previous Table.

Column 1 indicates the nature of the scattering material, Column 2 the angle of scattering, and Column 3 the value of HR in gauss-cm. Column 4 gives the corresponding energy of the individual β-particles, E, measured in volts, that is to say, their kinetic energy, T, divided by the charge, *e*, on the particles. Column 5 gives the values of t_m, the thickness which would be required to reduce the radiation through the stop to half value. Column 6 contains the corresponding values of $\phi/\sqrt{t_m}$, and Column 10 the values of $\mathrm{E}\phi/\sqrt{t_m}$. The remaining columns in the Table will be referred to later.

The Nuclear Theory of Scattering.

The deflection experienced by a particle passing in the neighbourhood of another fixed particle, which attracts or repels it with a force varying inversely as the square of the distance, can easily be solved by ordinary dynamical methods. Thus, if the nucleus of the atom can be regarded as acting as a point-charge of magnitude E_1, Rutherford showed that a particle carrying a charge, E_2, and moving with a kinetic energy, T, would be deflected through an angle, ϕ, given by the relation

$$p = \frac{\mathrm{E}_1\mathrm{E}_2}{2\mathrm{T}} \cot\frac{\phi}{2} \tag{6}$$

where p is the perpendicular distance of the centre of the nucleus from the original direction of the moving particle.

If t is the thickness of the scattering material, and n the number of atoms per unit volume, the probability that the path of a single moving particle will pass within a distance, p, of any nucleus is πntp^2, if t is small. The probability that it will not pass within this distance is therefore $1-\pi ntp^2$, which is also the probability that its deflection will be less than ϕ.

Hence, the fraction of the particles which are scattered through an angle less than ϕ, is given by

$$I/I_0 = 1-\pi ntp^2 = 1-\pi nt\left(\frac{E_1E_2}{2T}\right)^2\cot^2\frac{\phi}{2}. \tag{7}$$

This formula has been completely verified by Chadwick in the case of the scattering of the α-particles by silver, copper, and gold, and the charge E_1 on the nucleus has been shown to be equal to Ne, where N is the atomic number of the scattering element, to an accuracy of approximately 1 per cent.

Rutherford suggested that the same considerations should apply also to the scattering of the β-particles. In this case the charge E_2 is simply e, and, putting $E_1 = Ne$, the radiation passing through a stop of angle ϕ should be given by

$$I/I_0 = 1-\pi nt\frac{N^2e^4}{4T^2}\cot^2\frac{\phi}{2}. \tag{8}$$

We have already seen that the relation between I/I_0 and t is of this form.

The theory, however, requires some modification before it can be applied to the scattering of β-particles of the velocity used in the present experiments. The relation (6) was obtained on the assumptions that the mass of the particle remained constant, and that its energy was measured by $\frac{1}{2}mv^2$. These assumptions are sufficiently correct in the case of the α-particles, but are not applicable to β-particles, which, commencing with a velocity of over two-thirds that of light, travel with ever increasing speed as they approach the attracting nucleus. This has already been pointed out by Darwin* in connection with a theory of the absorption of the β-rays. For close distances of approach, the rapid increase in mass of the particles will cause them to move more slowly than if the mass had been constant. They are thus subjected to the attractive force of the nucleus for a longer time than would otherwise have been the case, and the deflection is thus increased. Darwin showed that, within a certain critical distance, the particle would actually be deflected into the nucleus, thus causing an absorption of the rays.

The relations obtained in the paper referred to were not immediately applicable to the present experiments. Mr. Darwin has very kindly worked out for us the complete theory, and has shown that, if p is the perpendicular distance from the nucleus of the undeflected path of the particle, and ϕ the corresponding deflection, the latter is given by the relation

$$\beta\cot\psi = \tan\left\{\pi-(\cos\psi)\frac{\phi+\pi}{2}\right\} \tag{9}$$

where $\operatorname{cosec}\psi = p/p_0$ and $p_0 = \dfrac{Ne^2}{c^2}\dfrac{\sqrt{1-\beta^2}}{\beta}$.

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* C. G. Darwin, 'Phil. Mag.' [6], vol. 25, p. 201 (1913).

This equation replaces equation (6) of the simple theory, and (7) thus becomes

$$\frac{\mathrm{I}}{\mathrm{I}_0} = 1-\pi ntp^2 \tag{10}$$

where p is now deduced from equation (9).

Probably the simplest way of comparing the experimental results with the theory is to regard the relations given by (9) as providing a correcting term to be applied to the simpler form of the theory. Let p' be the distance of approach necessary to produce a deflection, ϕ, when the variation of mass with velocity is neglected, then $p' = (\mathrm{N}e^2/2\mathrm{T})\cot\phi/2 = \mathrm{N}e^2/\mathrm{T}\phi$ to an accuracy of about 0·2 per cent., since the maximum value of ϕ in these experiments is about 8°. We may thus write (10) in the form

$$\mathrm{I}/\mathrm{I}_0 = 1-\pi nt\frac{\mathrm{N}^2e^4}{\mathrm{T}^2\phi^2}\left(\frac{p}{p_1}\right)^2 \tag{11}$$

where

$$\left[\frac{p}{p_1}\right]^2 = \frac{\operatorname{cosec}^2\psi}{\{\beta^2/4(1-\sqrt{1-\beta^2})^2\}\cot^2\phi/2} = f^2(\phi, \beta),$$

a function of ϕ and β only, which can be determined by (9).

If t_m is the thickness of scattering material necessary to reduce the radiation through a stop, whose radius subtends an angle ϕ at the substance, to half value

$$\pi nt_m\frac{\mathrm{N}^2e^4}{\mathrm{T}^2\phi^2}f^2(\phi, \beta) = \tfrac{1}{2}. \tag{12}$$

$f^2(\phi, \beta)$ thus represents the correction which, on the present form of the relativity theory, should be applied to the simpler theory to allow for variations in the mass of the β-particle. Values of the function under various experimental conditions are given in Table III, Column 7. In view of possible developments or modifications of the theory, however, we have thought it advisable to record in every case the actual "uncorrected" measurements, leaving the effect of applying the correcting factor to be discussed separately.

Variation of t_m with the Angle of Scattering.

The values of ϕ^2/t_m are given in Column V of Table II. It will be seen that to a first approximation the ratio is a constant for a given substance over the range of angles included in the Table. There is, however, a distinct difference to be noticed between the results for an element of low atomic weight, such as aluminium, and that for an element of high atomic weight, such as gold. The values of ϕ/t_m for the former are constant within the limits of experimental error; but those for gold show a small but distinct increase as the angle of the stop in increased. We shall show later that this increase becomes rapid for angles somewhat greater than those covered in the present experiments.

On the more complete theory represented by (12), the value of $\phi^2/t_m f^2(\phi, \beta)$ should be constant for a given substance. The correcting factor $f^2(\phi, \beta)$ varies very slowly with ϕ over the range of angles included in the Table, and the correction only amounts to a few per cent. In the case of aluminium and copper the corrected values are distinctly less constant than the uncorrected values. The variations are, however, too small to provide a satisfactory test of the theory, and experiments over a wider range of angles will be required before definite conclusions can be drawn on this point.

Variation of the Scattering with the Energy of the Rays.

The deflection produced in a β-particle during a single encounter varies with the energy of the particle in a manner which depends on the law of force governing the encounter. Neglecting the variation of mass with velocity, the value of $\phi/\sqrt{t_m}$ should be inversely proportional to the energy of the particle, if the force obeys the inverse square law, and inversely as the square root of the energy, if the law is that of the inverse cube of the distance. The former assumption corresponds to ordinary electrostatic action, the latter to magnetic action.

The problem is, however, complicated by the fact that the mass of the β-particle varies rapidly with its velocity, especially if the collision is a close one, and the correction to be applied for this variation is not as certain as might be desired. If, however, we accept (12) as giving the correct relation on the inverse square law of force, then $\phi/\sqrt{t_m} f(\phi, \beta)$ should be inversely proportional to the energy, or $(\phi/\sqrt{t_m})\,E/f(\phi, \beta)$ should be constant, the value of E being deduced from the magnetic deflection $H\rho$ by the relation (5).

Experiments were made with rays varying in energy from $2{\cdot}11 \times 10^5$ "volts" to $1{\cdot}03 \times 10^6$ "volts," using gold, aluminium and carbon as the scattering substances. The results are given in Table III. The actual experimental data will be found in Columns 3 and 5; Column 3 giving the value of $H\rho$ for the rays employed and 5 the number of equal foils required to cut down the radiation to half value. The value of $\phi/\sqrt{t_m}$ is contained in column 6.

Column 4 gives the energy of the particles in volts deduced from (5). The corresponding values of the correcting factor $f^2(\phi, \beta)$, are given in Column 7. The values of $\phi/\sqrt{t_m}\,.\,E$ are contained in Column 10 of the Table, and the values of $\phi/\sqrt{t_m}\,.\,E/f(\phi, \beta)$ in Column 11. The numbers in this column should therefore be constant on the present theories, if the law is that of the inverse squares.

The measurements for rays of very high and very low velocity are difficult owing to the small amount of such radiation emitted by the emanation tube. The values for gold must be regarded as very satisfactory under the circumstances, and may be taken as sensibly constant over the range of velocities employed, and at the particular angles employed. It is doubtful, however, whether the same can be claimed for the results for aluminium and carbon. For these elements the ratio does appear to increase steadily with increase in energy in each case, and the variation seems to be distinctly greater than the possible errors of experiment, as indicated by the experiments for gold. This would indicate either than the theory from which the correcting factor in Column 7 is calculated is inadequate in the case of these light atoms, or that the law of force in these cases is not accurately that of the inverse squares. As we shall show later, the collisions in the case of light elements are on an average much closer than those for a heavy element such as gold.

As a matter of interest the values of $\frac{1}{2}mv^2/e$, where m is the mass of the β-particle calculated from the Lorentz equation, are tabulated in Column 8, and the values of $\phi/\sqrt{t_m}\,.\,\frac{1}{2}mv^2/e$ in Column 9. The agreement of the figures in Column 9 is less close than that of the figures in Column 11.

Variation of the Scattering with the Atomic Number.

It will be convenient, in discussing the relation between the scattering and the atomic number of the element producing it, to write the equations in a slightly different form. Let M be the mass of a hydrogen atom ($= 1{\cdot}65 \times 10^{-24}$ grm.) and A the atomic weight of the scattering substance. The number of atoms n per unit volume of the substance is therefore Δ/MA, where Δ is the density of the substance. Now, if x_m is the mass per unit area of the foil required to cut down the radiation to half value, $x_m = \Delta t_m$, whence $n\,.\,t_m = x_m/\mathrm{AM}$. Substituting this value for $n\,.\,t$ in the scattering relations (12) and (8), we see that, in either case, for the same stop, and for the same velocity of the radiation, $\phi^2\mathrm{A}/x_m$ should be directly proportional to N^2, the square of the atomic number of the element, since $f(\phi, \beta)$ depends only on ϕ and β. Thus, on either form of the theory, $(\phi^2/x_m)\,\mathrm{A}/\mathrm{N}^2$ should be constant. The last column of Table II, which contains the experimental values of this quantity, shows that this is very far from being the case, the scattering by the lighter elements being considerably greater than that produced by the heavier elements.

This is shown graphically by the full curve in fig. 5, in which the experimental values for $\phi\sqrt{\mathrm{A}/x_m}$ have been plotted against the atomic number. It will be seen that the departure from a linear relation is very considerable. There is thus a distinct discrepancy between the experimental results and the nuclear theory of scattering in its present form, and this discrepancy is probably the most important point which emerges from the present experiments.

The nature and extent of the divergence may be best illustrated by evaluating the theoretical equations for scattering (which contain no unknown constants), and comparing the results with the experimental values. So far we have neglected the effect of the electrons in the atom on the paths of the β-particles. It is easy to show that the effect should be relatively small. Assuming that the electron in the atom behaves as a fixed point charge of magnitude e, then in (6) $E_2 = E_1 = e$. Also, if N is the atomic number of the element, the total number of electrons per unit volume is Nn, and the fraction of the β-particles scattered through an

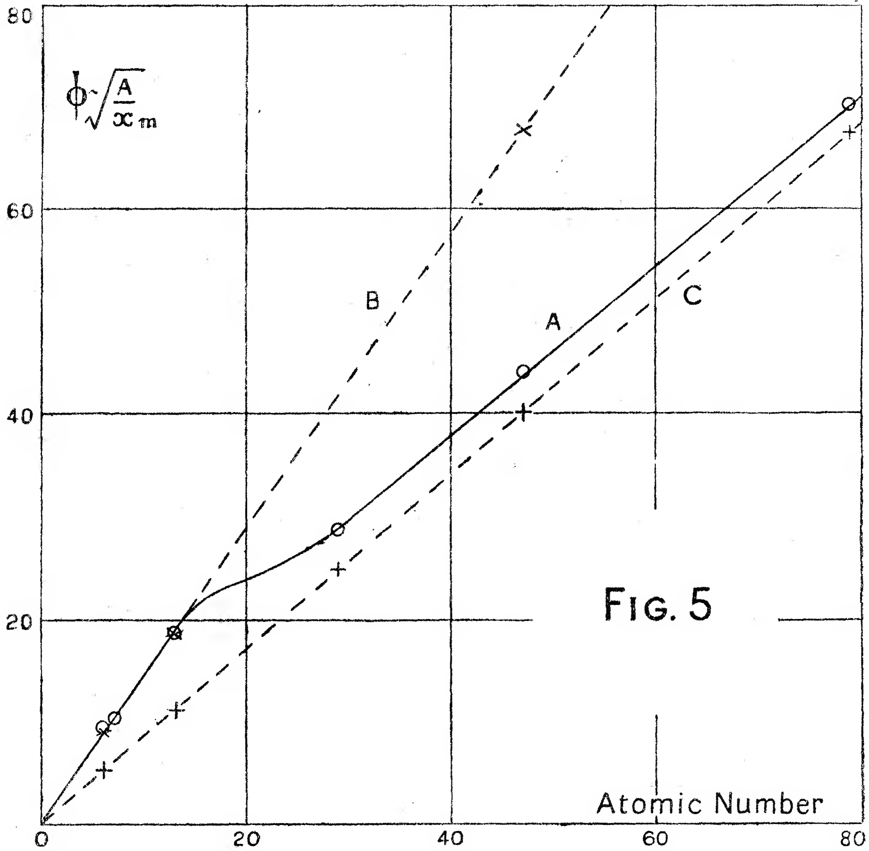

FIG. 5

Variation of Scattering with Atomic Number.

A, present series, ϕ = 0·145 radians ; B, Crowther (previous series), ϕ = 0·314 radians ; C, theoretical values.

angle greater than ϕ by collision with the electrons in the atoms is thus $\pi . \mathrm{N}nt(e^2/2\mathrm{T})^2 \cot^2 \phi/2$. The fraction scattered outside the same angle by the nuclear collisions is $\pi nt(\mathrm{N}e^2/2\mathrm{T})^2 \cot^2 \phi/2$. Thus, taking into account both form of scattering, equation (8) becomes

$$\frac{\mathrm{I}}{\mathrm{I}_0} = 1 - \pi nt \frac{e^4}{4\mathrm{T}^2} (\mathrm{N}^2 + \mathrm{N}) \cot^2 \phi/2. \tag{13}$$

The correction for electron scattering amounts to about 16 per cent. in the case of carbon, 7 per cent. for aluminium, and is negligible for the other elements investigated.

Thus, taking into account the variation of mass with velocity, (12) now becomes

$$\pi nt t_m \frac{e^4}{\mathrm{T}^2\phi^2} (\mathrm{N}^2 + \mathrm{N}) f^2(\phi, \beta) = \tfrac{1}{2} \tag{14}$$

which now includes both the nuclear and electronic scattering. This can easily be evaluated by writing $nt = x_m/\mathrm{AM}$ as before. The values of $\phi\sqrt{\mathrm{A}/x_m}$, deduced from this equation, are given in Column 3 of Table IV, the experimental values being repeated, for the sake of comparison, in Column 2.

Table IV.

Element.	$\phi\sqrt{\mathrm{A}/x_m}$ (observed).	$\phi\sqrt{\mathrm{A}/x_m}$ (theoretical).	k.
Gold	70·0	67·2	8·6
Silver	44·2	40·1	11·0
Copper	28·3	24·9	9·7
Aluminium	18·6	11·4	24·3
Carbon	9·9	5·4	17·0

It will be seen that in all cases the scattering actually observed is greater than that to be expected on the theory. The divergence is greatest in the case of the lighter elements, and decreases with increasing atomic number. Even in the case of gold, however, there is still a difference of about 5 per cent. between the two values. As the actually observed quantity is ϕ^2/x_m, this would require an error in the observations of practically 10 per cent. The theoretical values are also indicated by the lower (broken) curve in fig. 5.

Discussion of Results.

It seems clear, from the figures and calculations of the previous section, that our present theories are inadequate to account for the observed scattering of a beam of β-particles during its passage through matter.

As the nature of the electrical force exerted by the nucleus seemed to have been so firmly established by experiments on the scattering of α-particles, it was perhaps inevitable to consider whether, for some unknown reason, the deflections due to the electrons in the atom had been under-estimated. The theory that the electron is also a magneton, though not yet orthodox science, lends colour to the suggestion that in a collision between two electrons magnetic forces might be called into play which might result in a deflection considerably greater than that due to their electric charges. The relation between scattering and atomic number would then be of the form $c(N^2+kN)$ instead of $c(N^2+N)$ where k would be some constant greater than unity.

It is very doubtful whether a relation of this form is capable of representing the experimental results, which are shown graphically in the full curve in fig. 5. It is, of course, possible to choose a value for k which will give a smooth curve running fairly evenly among the experimental points on the graph. If we assume that the constant c has the value given by (12) the corresponding value of k can be calculated from the experimental data for the different elements. The results are given in Table IV. Taking 14 as an approximate mean value for k the calculated values for the scattering would agree with the observed values, to within the limits of experimental error, for all the elements except aluminium. For this element, however, the error in $\phi/\sqrt{x_m}$ would amount to 10 per cent., implying an error in the actual observed value for x_m of as much as 20 per cent. As the value for aluminium is the one which has been determined with the greatest accuracy, and is the mean of a considerable number of closely concordant observations, it does not seem possible that an error of this magnitude can exist. Unless the scattering by aluminium is anomalous we must conclude that a formula of the type suggested cannot express the whole of the observations. It may, however, be worth noting, in this connection, that aluminium is the only one of the scattering substances employed which is paramagnetic, the magnetism being further increased in the case of the commercial metal by the appreciable amount of iron (usually about 1 per cent.) which it invariably contains.

It may be pointed out that if the value of k is as large as is required to account for the experimental values, the force between two electrons must be considerably greater than that given by Coulomb's law. Now with this value of k the scattering for the lighter elements would be mainly electronic, while that of gold would still be almost entirely nuclear. Thus, unless the law of force between the β-particle and an electron were the same as that between the β-particle and a nucleus, we should expect that the relation between the scattering and the energy of the β-particles should differ for light and for heavy elements. The results given in Column 11 of Table III do not absolutely rule out this suggestion; but the observed variations are much less than would be expected if the forces were mainly magnetic.

Compton* has recently published a magnetic theory of the deflection of the β-particles. From a study of C. T. R. Wilson's very beautiful pictures of the actual tracks of β-particles through air, he concludes that the deflections of the particles are not random in direction, but that each particle has a kind of bias, which results in its deflections being all in the same direction. This is explained as being due to induced magnetism, produced in the scattering substance during the passage of the particle through it. The treatment in the original paper is obviously inadequate, being based on macroscopic considerations, whereas from the point of view of a β-particle the scattering substance is merely a space very sparsely studded with positive and negative charges. The author promises a reconsideration of the subject from this aspect, which he believes will lead to the same results; but this is not available at the time of writing.

The magnetic scattering suggested by Compton and others is clearly a multiple scattering. In fact, in its present form, it consists of a gradual bending of the track of the particles, due to a continuous force acting upon it. In its atomic form it would presumably be due to a large number of small deflections. If these were random in direction, the relation between the transmitted radiation and the thickness of scattering material would be given by an equation of the form of (3). In Compton's theory the deflections are all in the same direction. The form of the curve under these circumstances can easily be calculated, but it is obvious that, for the same value of t_m, the initial flattening of the curve would be even more pronounced than if the displacements were random. The shape of the experimental curves of figs. 2 and 3 make it clear that we are not dealing in the present experiments with inflections of this type.

Comparison with Previous Results.

It may be interesting at this stage to compare the results obtained in the previous paper† with the present experimental and theoretical values. A little calculation shows that the finite angle of the beam used in the previous work should not produce any very serious error in the values obtained for ϕ^2/t_m. The velocity of the radiation used was rather greater than in the present experiments, and the angle of the stop was considerably larger (18° as against 8° in the present work). We can, however, compare them with the present series by making use of the relations already discussed in previous sections. The values so obtained are given in Column II of Table V. It will be noticed that the values for carbon and aluminium are in excellent agreement with those obtained in the present experiments. This close agreement in the case of two elements makes it difficult to believe that the results obtained under identical conditions for the remaining elements can be seriously in error. Experiments now in progress, using stops of larger aperture, are, in fact, giving results for the heavier elements which confirm the general accuracy of the previous results.

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* 'Phil. Mag.,' 1920.

† J. A. Crowther, 'Roy. Soc. Proc.,' A, vol. 84, p. 226 (1910).

Table V.

I.	II.	III.
Element.	$\phi/\sqrt{t_m}$ (old series).	$\phi/\sqrt{t_m}$ (new series).
Carbon	2·7	2·9
Al	5·8	5·9
Ag	20·8	13·6
Au	—	21·7
Pt	39·3	

Values given for energy of 4·59 . 10^5 volts.

It will be seen, however, that the results for the heavier elements are considerably greater than those obtained in the present experiments. In fact the relation between $\phi\sqrt{A/x_m}$ and N (indicated by the upper broken curve in fig. 5) is almost linear for the whole of the elements investigated.

General Conclusions.

Taking into account both series of experiments the results may be summarised by saying that, for light elements such as carbon and aluminium, the scattering as measured by $\phi/\sqrt{t_m}$ is independent of ϕ over the whole range of angles investigated (*i.e.*, from 4° to 18°) but has a value which is nearly twice that to be expected on the current theories of the effect. On the other hand for heavy elements, such as gold or platinum, the scattering for very small values of ϕ approximates closely to that to be expected from theoretical considerations, but increases rapidly with the angle, until for angles of 18° it is in agreement with the larger scattering shown at all the angles measured by the lighter elements. This is the main result which emerges from the observations.

The scattering of β-particles has so far been investigated only over a limited range both of angles and velocities, and it is clear that, in view of the unexpected nature of the results, a much more extensive survey of the subject is required and this is now in progress. Attention may, however, be called to following points.

The distance p (see (6)) within which the original path of the particle must approach the centre of the deflecting nucleus in order to be deflected through an angle greater than ϕ, decreases as the angle is increased. Thus the particles deflected out of the emerging beam will have made closer and closer approaches to the nucleus as the angle of the limiting stop is increased. The earlier experiments, therefore, refer to β-particles which have, on the whole, been much closer to the deflecting particles than those dealt with in the present series. The value of p in any given case can easily be deduced from the thickness of the scattering material. Thus, taking the values of t_m corresponding to the largest stop used in the present experiments, we find that for aluminium the undeflected paths of half the particles must have passed within a distance of $6{\cdot}5 \times 10^{-11}$ cm. of a nucleus. The corresponding value of p for gold is $2{\cdot}5 \times 10^{-10}$ cm. The collisions are, therefore, much closer for aluminium than for gold. In the earlier experiments, in which owing to the larger angle of stop, the values of t_m are greater, the average value of p for gold was approximately one-half that in the present series, and the paths of half the β-particles had approached within a distance of $1{\cdot}2 \times 10^{-10}$ cm. of a nucleus. Thus the results show that the scattering by gold changes from a value approximating closely to that given by the theory to the higher value corresponding to aluminium, when the average value of p changes from $2{\cdot}5 \times 10^{-10}$ cm. to $1{\cdot}2 \times 10^{-10}$ cm. The average values of p for aluminium are smaller than the latter value for all the angles investigated.

This method of viewing the data strongly suggests that some change is required in the present theory when the distance between the path of the β-particle and the deflecting particle is less than a certain critical value. Similar considerations (but with different constants) obviously also apply if we consider the collisions as taking place between the β-particle and the electrons in the atom. It is possible that the theory represented by the relation (9), for some reason not yet clear, does not adequately represent the motion of a β-particle which approaches very near to an atomic nucleus, acquiring in the process a velocity approaching within a very small fraction of the velocity of light. It is also possible that there may be some modification of the field at a certain distance from a nucleus or an electron, so as to produce a larger deflection of the β-particle than would be expected on the present theories. These suggestions are also supported by the results obtained on the variation of the scattering with the energy of the rays, where again the results were in much better agreement with the theory for gold, than for the lighter elements, where the collisions on an average were much closer. It is hoped that the further survey now being made may indicate more clearly the nature of the modifications required.

We wish to express our warmest thanks to Prof. Sir Ernest Rutherford for his continued interest in these experiments and for his kindness in supplying the considerable quantities of emanation employed in the work. We are also indebted to Mr. Crowe for constructing and filling the emanation tubes.

Boundary Lubrication.—The Paraffin Series.

By W. B. Hardy, Sec. R.S., and Ida Doubleday.

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(A Report to the Lubrication Committee of the Department of Scientific and Industrial Research.)

In what is often called complete lubrication, the kind of lubrication investigated by Towers and Osborne Reynolds, the solid surfaces are completely floated apart by the lubricant. There is, however, another kind of lubrication in which the solid faces are near enough together to influence directly the physical properties of the lubricant. This is the condition found with "dry" or "greasy" surfaces. What Osborne Reynolds calls "boundary conditions" then operate, and the friction depends not only on the lubricant, but also on the chemical nature of the solid boundaries. Boundary lubrication differs so greatly from complete lubrication as to suggest that there is a discontinuity between the two states. In the former the surfaces have the property of static friction, and the resistance is some inverse function of the viscosity of the lubricant. In complete lubrication static friction is absent and the resistance varies directly with the viscosity of the lubricant. Boundary lubrication is alone considered in this paper.

The enquiry is limited to the lubricating qualities of normal paraffins and their related acids and alcohols. The molecules of the substances employed, therefore, consist either of a simple chain of carbon atoms to which are attached atoms of hydrogen, or of such a chain loaded at one end with the hydroxyl group—OH, or the carboxyl group—COOH.

Attention was concentrated chiefly on three variables—the quantity of lubricant present, the composition of the solid faces, and the chemical constitution of the lubricant. The relations disclosed by the experiments are of surprising simplicity, whilst their interpretation is difficult. For this